On Approximating (Sparse) Covering Integer Programs

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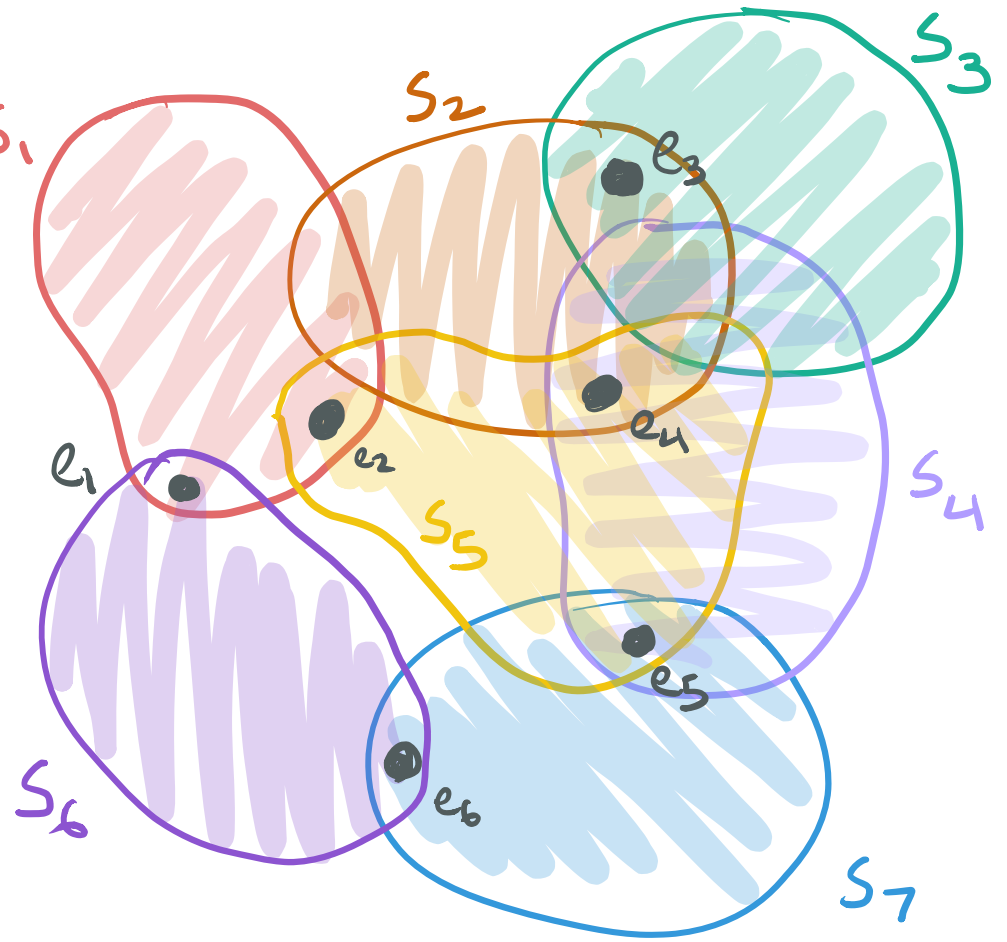
Set cover

Input: sets $S_1, \dots, S_n \subseteq \{e_1, \dots, e_m\}$

[n sets
m elements]

Goal: smallest collection $S_{i_1}, \dots, S_{i_k}$ covering $\{e_1, \dots, e_m\}$

$$\bigcup_{j=1}^{k} S_{i_j} = \{e_1, \dots, e_m\}$$



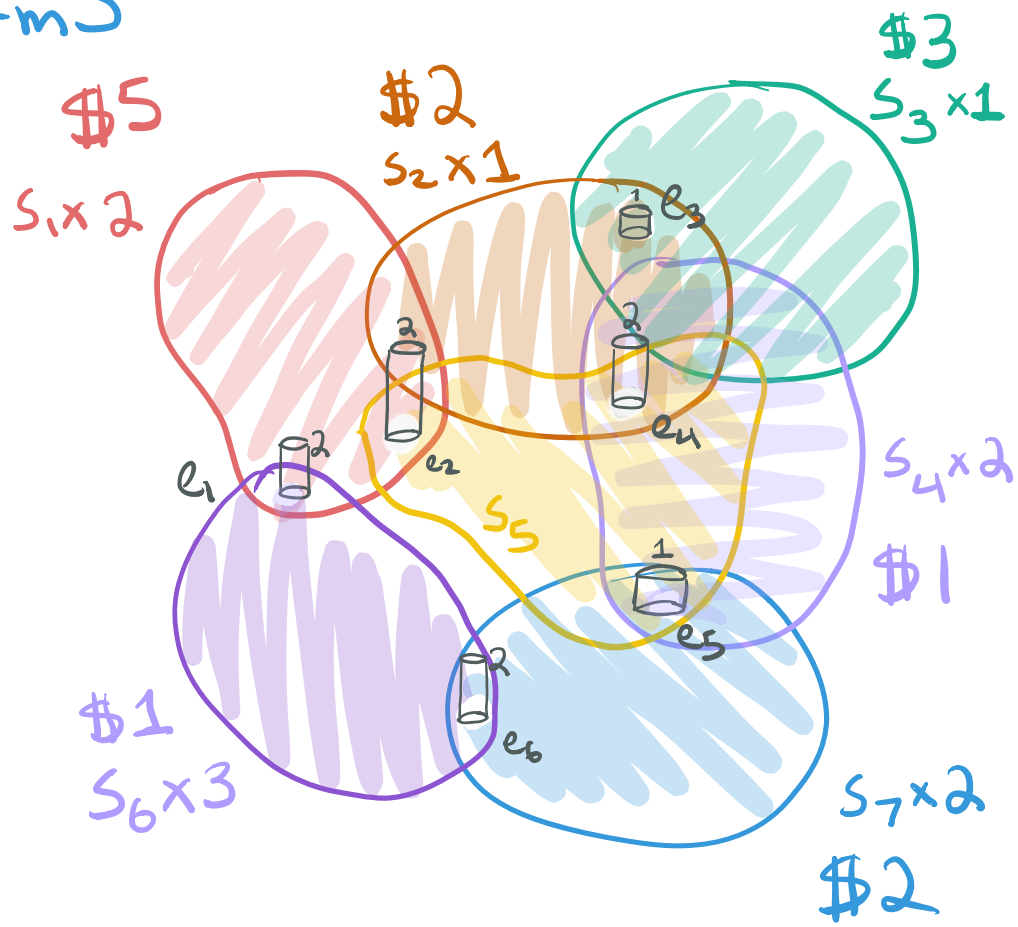
- greedy, randomized rounding => ln m APX

↷ Tight unless P=NP

Multiset Multicover

Input: sets $S_1, \ldots, S_n \subseteq \{e_1, \ldots, e_m\}$

+ costs $c \in \mathbb{R}^n_{>0}$ (on sets)
+ demands $b \in \mathbb{Z}^m_{>0}$ (on elem.)
+ multiplicities $d \in \mathbb{Z}^n_{>0}$ (on sets)



Goal: min cost collection (w/ mult. up to d) s.t. each e_j is covered $\geq b_j$ times

Covering Integer Programs (CIPs)

minimize $\langle c, x \rangle$ over $x \in \mathbb{Z}^n_{\geq 0}$ (cost)

s.t. $Ax \geq b$ (covering constraints)

(optionally) $x \leq d$ (multiplicity constraints)

$A \in [0,1]^{m \times n}$ $b \in \mathbb{R}^m_{\geq 0}$ $c \in \mathbb{R}^n_{\geq 0}$ $d \in \mathbb{R}^n_{\geq 0}$

minimize $\langle c, x \rangle$ over $x \in \mathbb{Z}^n_{\geq 0}$ (cost)

s.t. $Ax \geq b$ (covering constraints)

(optionally) $x \leq d$ (multiplicity constraints)

Q: How well and how fast can we approximate CIP's?

(our contributions:

- improved APX w/r/t "column sparsity"
- nearly-linear time APX for underlying LP)

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Q: How well and how fast can we approximate CIP's?

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Overview for the rest of the talk

① BACKGROUND: • randomized rounding
- knapsack cover inequalities • sparsity parameters
- Rounding for sparse CIPs

② FORMAL STATEMENT OF RESULTS

③ OVERVIEW OF ROUNDING SCHEME
(hopefully intuitive)

Approximating CIP's w/out multiplicities "CIP_∞"

Raghavan, Thompson (1980s)

① solve the LP $\Rightarrow x \in \mathbb{R}^m_{\geq 0}$ s.t. $\langle c, x \rangle \leq OPT$, $Ax \geq b$

② scale up solution by $\alpha = O(\ln m)$ $\Rightarrow y = \alpha x \in \mathbb{R}^m_{\geq 0}$

③ randomly & independently round each coordinate up/down to integers $\Rightarrow z \in \mathbb{Z}^m_{\geq 0}$ s.t. $E[z] = y$

$\Rightarrow O(\ln m)$ APX for CIP_∞

Approximating CIP's (w/ multiplicities $d \in \mathbb{Z}^n_{\geq 0}$)

[Kolliopoulos, Young '00]

① solve the LP w/ knapsack cover inequalities $\Rightarrow x \in \mathbb{R}^m_{\geq 0}$ s.t. $\langle c, x \rangle \leq OPT$, $Ax \geq b$

② scale up solution by $\alpha = O(\ln m)$ and truncate $y_j = \min\{\alpha x_j, d_j\}$

③ randomly & independently round each coordinate up/down to integers $z \in \mathbb{Z}^m_{\geq 0}$ s.t. $E[z] = y$

$\Rightarrow O(\ln m)$ APX for CIP

Knapsack Cover (KC) inequalities

[Carr, Fleischer, Leung, Phillips SODA 2000]

$$\min \langle c, x \rangle \quad Ax \geq b \quad x \leq d \quad x \geq 0$$

$\Rightarrow$

$$\min \langle c, x \rangle \quad A_S x \geq b_S \text{ for all } S \subseteq [n] \quad x \geq 0$$

where for $S \subseteq [n]$,

$$b_{S,i} = \max \{0, b_i - \sum_{j \in S} A_{ij} d_j\}$$

$$A_{S,ij} = \begin{cases} 0 & \text{if } j \in S \\ \min\{A_{ij}, b_{S,i}\} & \text{if } j \notin S \end{cases}$$

Intuitively, $A_S x \geq b_S$ is the residual set cover problem after taking all coordinates in S

Solving Knapsack Cover (KC) inequalities

$$\min \langle c, x \rangle$$

$$A_S x \geq b_S \text{ for all } S \subseteq [n]$$

$$x \geq 0$$

Carr, Fleischer, Leung, Phillips [SODA 2000]

- give separation oracle for ellipsoid
- $(1+\varepsilon)$-APX in $\tilde{O}(nN \log(C) \text{poly}(1/\varepsilon))$

via MWU, where $C = (\max A_{ij})/(\min A_{ij})$

N = nonzeroes in A

Approximating CIP's (w/ multiplicities $d \in \mathbb{Z}^n_{\geq 0}$)

[Kolliopoulos, Young '00]

① solve the LP w/ knapsack cover inequalities $\Rightarrow x \in \mathbb{R}^m_{\geq 0}$ s.t. $\langle c, x \rangle \leq OPT$, $Ax \geq b$

② scale up solution by $\alpha = O(\ln m)$ and truncate

$y_j = \min\{\alpha x_j, d_j\}$

Apply KC inequalities to $S = \{j : y_j = d_j\}$

③ randomly & independently round each coordinate up/down to integers

$z \in \mathbb{Z}^m_{\geq 0}$ s.t. $E[z] = y$

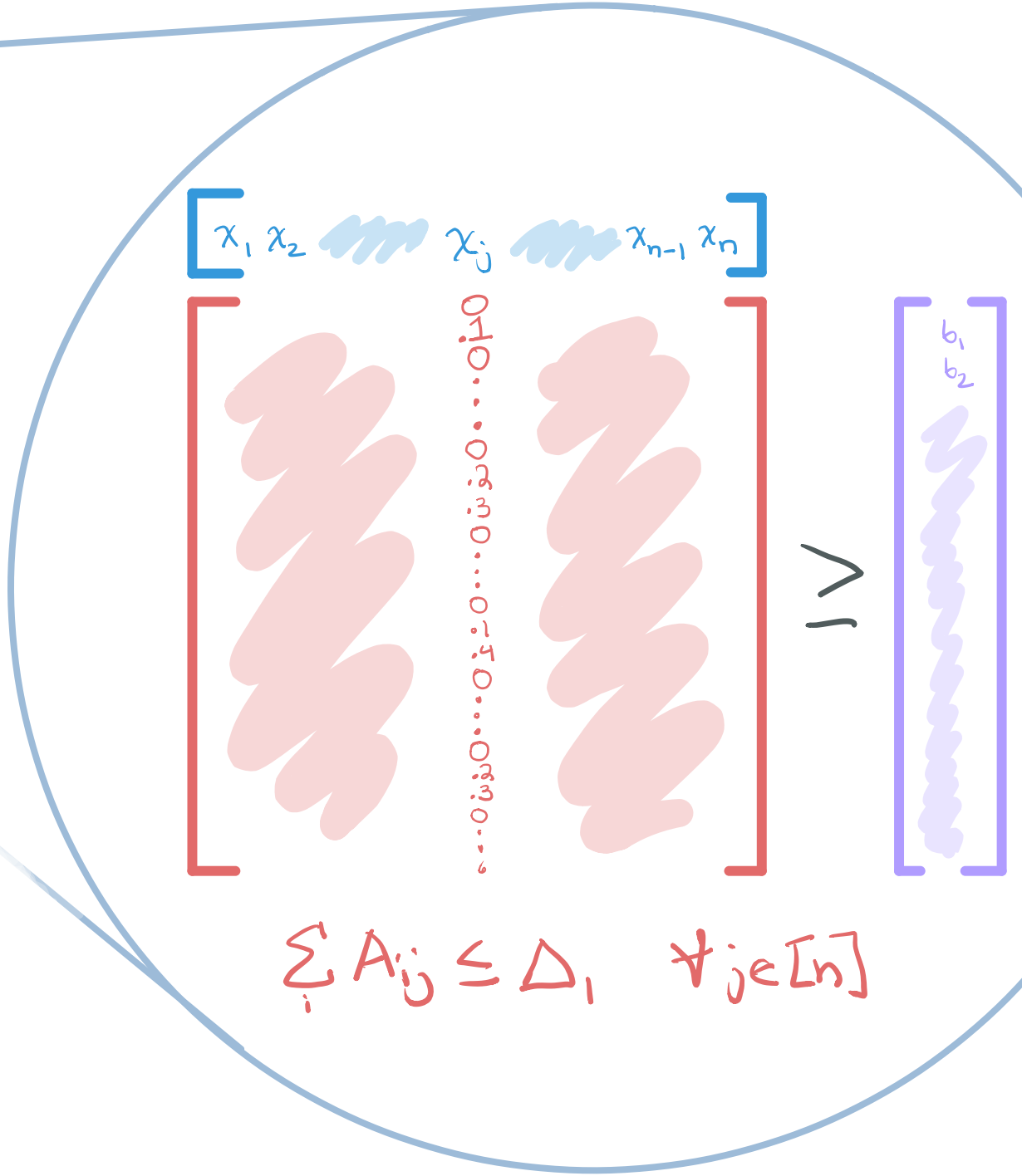
$\Rightarrow O(\ln m)$ APX for CIP

Column Sparsity

$Ax \geq b$

Δ_0 = max # of nonzeroes in a column

Δ_1 = max sum of entries in any column



Why Δ_1?

$$\Delta_1 \leq \Delta_0 \leq m$$

compared to Δ_0,

Δ_1 is robust to noise

Approximating CIPs w/r/t column sparsity (abbr.)

[Srinivasan '99] $O(\log \Delta_0)$ for CIP_∞

[Kolliopoulos, Young '00] $O(\log \Delta_0)$ for CIP w/ KC inequalities

{

[Chen, Harris, Srinivasan SODA 2016]

- tight bounds
- Δ_1 type bounds (!!!)

[Chen, Harris, Srinivasan SODA 2016]

Rounds LP by "resampling" technique inspired by Moser-Tardos LLL framework

CIP: $\ln \Delta_0 + O(\sqrt{\ln \Delta_0})$ * via KC inequalities

CIP_∞: $\ln \Delta_1 + O(\sqrt{\ln \Delta_1})$ ** via basic LP

Bicriteria CIP: returns $z \in \mathbb{Z}^n_{\geq 0}$ w/ mult. $z \leq (1+\varepsilon) d$,

cost $\left(\frac{\ln \Delta_1}{\varepsilon} + O\left(\sqrt{\frac{\ln \Delta}{\varepsilon}}\right)\right) OPT$ ** via basic LP

* ignoring dependency on $\min_i b_i$ for simplicity

*At time of our work; recently improved (see latest Arxiv)

Our Results: 2 flavors

① Faster APX LP solver for KC inequalities

② New APX bounds

Theorem:

Let $\varepsilon > 0$ be fixed. Given $\longrightarrow$

$$\min \langle c, x \rangle \quad Ax \geq b \quad x \leq d \quad x \geq 0$$

in $\tilde{O}\left(\frac{N \log C}{\varepsilon^3} + \frac{(m+n)\log C}{\varepsilon^5}\right)^*$ deterministic time, one can compute $x \in \mathbb{R}^n_{\geq 0}$ s.t. $\langle c, x \rangle \leq OPT$, $x \leq d$, and

$$A_S x \geq (1-\varepsilon) b_S \quad \forall S \subseteq [n].$$

- Accelerates [Carr et al]'s MWU approach w/ dynamic Knapsack cover data structure

* N = nonzeroes, $C = \max A_{ij} \Big/ \min_{i,j : A_{ij} \neq 0} A_{ij}$

New APX bounds

CIP	CIP_∞	Bicriteria CIP ($z \le (1+\varepsilon)d$)
$\ln \Delta_0 + \ln\ln \Delta_0 + O(1)$	$\ln \Delta_1 + \ln\ln \Delta_1 + O(1)$	$\ln \Delta_1 + \ln\ln \Delta_1 + O(\ln(1/\varepsilon))$
via KC inequalities	via basic LP	via KC inequalities

* slightly improves 2nd order terms from $O(\sqrt{\ln \Delta})$ to $\ln\ln \Delta + O(1)$

* improves dependence on ε from multiplicative $\frac{1}{\varepsilon}$ to additive $\ln(1/\varepsilon)$

* Alteration rounding scheme: simple to analyze, derandomize

★ rest
★ of
★ talk

The algo: "round-and-fix" (for CIP_∞)

[Srinivasan '01] [Saket Sviridenko '12] [Gupta Nagarajan '16]

1. solve LP $\Rightarrow$ fractional solution $x \in \mathbb{R}^n_{\geq 0}$

2. (round) independently round $z_j \in \{\lceil \alpha x_j \rceil, \lfloor \alpha x_j \rfloor\}$

s.t. $E[z_j] = \alpha x_j$ for some $\alpha > 1$

3. (fix) for each constraint i s.t. $(Az)_i < b_i$

a. let $y^{(i)}$ solve 1-dim knapsack cover problem induced by ith constraint, set $z = z \vee y^{(i)}$

The algo: "round-and-fix" (for CIP_∞)

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(shorthand)

1. $x \leftarrow LP$
2. round $z \sim \alpha x$
3. fix $\{i : (Az)_i < b_i\}$

High-level analysis

$$E[\text{cost}] = \alpha \langle c, x \rangle + \sum_{i=1}^{m} \left\{ \Pr[(Az)_i < b_i] \times \text{fixing cost of constraint } i \right\}$$

Analysis

"round-and-fix"

1. $x \leftarrow LP$
2. round $z \sim \alpha x$
3. fix $\{i : (Az)_i < b_i\}$

$$E[\text{cost}] = \alpha \langle c, x \rangle + \sum_{i=1}^{m} \Pr[(Az)_i < b_i] [\text{fixing-cost}(i)]$$

Two regimes of interest

① $A_{ij} \in \{0, 1\}$; ie, $\Delta_0 = \Delta_1$

② $A_{ij} \in \{0, \varepsilon\}$ for very small $\varepsilon > 0$, ie $\Delta_1 = \varepsilon \Delta_0$

[Saket, Sviridenko]

(case 1): Δ_0 analysis

let $\alpha = O(\ln \Delta_0)$

1. $x \leftarrow LP$
2. round $z \sim \alpha x$
3. fix $\{i : (Az)_i < b_i\}$

- Chernoff:

$$\Pr[(Az)_i < b_i] \le O\left(\frac{1}{\Delta_0}\right)$$

- Fixing-cost(i):

(a) fix constraint i by letting $y_j^{(i)} = \begin{cases} x_j & \text{if } A_{ij} \neq 0 \\ 0 & \text{if } A_{ij} = 0 \end{cases}$

& integrality gap $= O(1)$

$$\sum_i \Pr[(Az)_i < b_i][\text{fix-cost}(i)]$$

$$= \frac{1}{\Delta_0} \sum_i \sum_{j : A_{ij} \neq 0} c_j x_j = \frac{1}{\Delta_0} \sum_j c_j x_j \sum_{i : A_{ij} \neq 0} 1 \le \sum_j c_j x_j$$

[Saket, Sviridenko]

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(case 2) $A_{ij} \in \{0, \varepsilon\}$ $\forall i,j$

let $\alpha = \ln(\Delta_1) + m$

1. $x \leftarrow LP$
2. round $z \sim \alpha x$
3. fix $\{i : (Az)_i < b_i\}$

- Chernoff amplifies exponentially:

$$Pr[(Az)_i \le b_i] \le \left(\frac{1}{\Delta_1}\right)^{1/\varepsilon}$$

- fix constraint i by $y_j^{(i)} = \{ x_j \text{ if } A_{ij} \neq 0,\ 0 \text{ if } A_{ij} = 0 \}$
  & int gap $= O(1)$

$$\sum_i Pr[(Az)_i < b_i][\text{fix-cost}(i)]$$

(case 2) $A_{ij} \in \{0, \varepsilon\}\ \forall i,j$

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$A_{ij} \in \{0,1\}$ ⟷ $A_{ij} \in \{0,\varepsilon\}$

$\Delta_0 = \Delta_1$ | $\Delta_0 = \varepsilon \Delta_1$

$\sum_i [\text{fix-cost}(i)] \approx \Delta_1 \langle c, x \rangle$ | $\sum_i [\text{fix-cost}(i)] \approx \frac{\Delta_1}{\varepsilon} \langle c, x \rangle$

$\Pr[(Ax)_i < b_i] \approx \frac{1}{\Delta_1}$ | $\Pr[(Ax)_i < b_i] \approx \left(\frac{1}{\Delta_1}\right)^{1/\varepsilon}$

Big A_{ij} => good fixing costs relative to Δ_1

small A_{ij} => exponentially smaller probability of fixing

How to get both?

the "median coefficient" ρ_i

choose ρ_i s.t. $\sum_{A_{ij} \geq \rho_i} A_{ij} x_j = \sum_{A_{ij} \leq \rho_i} A_{ij} x_j = \frac{b_i}{2}$

let $\alpha = \ln(\Delta_1) + \omega$

1. $x \leftarrow LP$
2. round $z \sim \alpha x$
3. fix $\{i : (Az)_i < b_i\}$

- Apply Chernoff to small coeffs:

$$Pr[(Az)_i < b_i] \approx \left(\frac{1}{\Delta_1}\right)^{1/\rho_i}$$

- fix constraint i by using big coefficients

$$\Rightarrow \sum_i [\text{fix-cost}(i)] \approx \sum_i \frac{\Delta_1}{\rho_i} \langle c, x \rangle$$

$$E[\text{fixing-cost}] \approx \sum_i \left(\frac{1}{\Delta_1}\right)^{1/\rho_i} \frac{\Delta_1}{\rho_i} \langle c, x \rangle \leq \langle c, x \rangle$$

the "median coefficient" ρ_i

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1. $x \leftarrow LP$
2. round $z \sim \alpha x$
3. fix $\{i : (Az)_i < b_i\}$

- Apply Chernoff to small coeffs:

$$Pr[(Az)_i < b_i] \lesssim \left(\frac{1}{\Delta_1}\right)^{1/\rho_i}$$

- fix constraint i by using big coefficients

$$\Rightarrow \sum_i [\text{fix-cost}(i)] \lesssim \langle c, x \rangle \sum_i \frac{\Delta_1}{\rho_i}$$

$$E[\text{fixing-cost}] \lesssim \sum_i \left(\frac{1}{\Delta_1}\right)^{1/\rho_i} \frac{\Delta_1}{\rho_i} \langle c, x \rangle \leq \langle c, x \rangle$$



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choose ρ_i s.t. $\sum_{A_{ij} \geq \rho_i} A_{ij} x_j = \sum_{A_{ij} \leq \rho_i} A_{ij} x_j = \frac{b_i}{2}$

let $\alpha = \ln(\Delta_1) + \omega$

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- Apply Chernoff to small coeffs:

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- fix constraint i by using big coefficients

$$\Rightarrow \sum_i [\text{fix-cost}(i)] \approx \langle c, x \rangle \sum_i \frac{\Delta_1}{\rho_i}$$

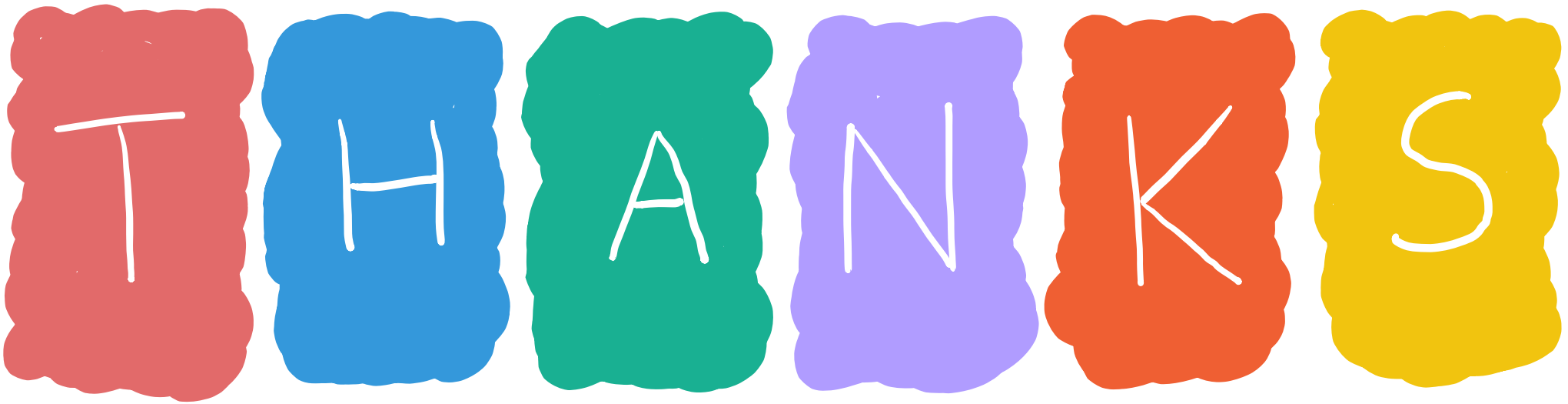
$$E[\text{fixing-cost}] \approx \langle c, x \rangle \sum_i \left(\frac{1}{\Delta_1}\right)^{1/\rho_i} \frac{\Delta_1}{\rho_i} \leq \langle c, x \rangle$$

Conclusion

① Nearly-linear time, deterministic APX for LP w/ Knapsack cover inequalities

② Tight(er), simple, derandomizable rounding algorithm for CIP's
"median coefficient trick"

① + ② = ③ Tight, deterministic nearly linear time approximation algorithm for CIP's.


THANKS